



George Washington Carver
High School of Engineering and Science
2020 Summer Enrichment

AP AB CALCULUS

Part 1: Exam grade worth 50 points (combined with 50 point exam for 100 point exam grade) due 9/9/2020 at the start of class.

This summer assignment requires you to review key precalculus and algebra topics so that you will not need to review all of these topics in depth during the school year. Included in this packet, you will find many questions on these concepts.

Your project is to complete all problems in detail. **SHOW ALL WORK.** You can do the work on notebook paper, on the packet, or both. When you do work on notebook paper, please clearly label the problem numbers. If there is no work to be shown (i.e. a multiple-choice problem that says "for which function is $f(-x) = -f(x)$ "...the answer is just whichever function listed is ODD...so you would just pick the odd fn...no work to show) then explain your **UNDERSTANDING** (i.e. exactly as I just did in the last set of parentheses.)

When school starts, we will spend a few days going over these problems. Afterwards, you will take a college placement exam (designed to assess your readiness for calculus.) The sum of your grades on this packet and the placement exam will be your grade for the summer project.

Rubric:

Item	Points Possible	Points Earned
All problems are correctly labeled and in order.	2	
Topic #1—Answers and Sufficient Work	4	
Topic #2—Answers and Sufficient Work (easily the longest section)	8	
Topic #3—Answers and Sufficient Work	4	
Topic #4—Answers and Sufficient Work	4	
Topic #5—Answers and Sufficient Work	4	
Topic #6—Answers and Sufficient Work	4	
Topic #7—Answers and Sufficient Work	4	
Topic #8—Answers and Sufficient Work	4	
Topic #9—Answers and Sufficient Work	4	
Topic #10—Answers and Sufficient Work	4	
Topic #11—Answers and Sufficient Work	4	
TOTALS (462Q)	50	

Part 2: College Board Account and AP Classroom (Homework Assignment Due Friday 9/4/2020 by 3PM)

A. You need to be able to access your College Board account. If you have multiple accounts, you need to combine them into one. This needs to be done because:

- Your AP tests will be ordered through your College Board account.
- College Board is releasing materials needed for class and this material will be accessed through your College Board account.
- You receive your scores through this account.

Due to privacy issues, E&S cannot help you with this process. You must call the College Board to fix any issues. You need to do this well before the first day of school. If you can't remember the email you used when you created your account or are still having trouble accessing your account, call College Board Customer Service at 866-315-6068. If the number changes over the summer, remember there is always Google.

B. Once you have your account, you will need to join the AP AB Class. You will receive a join code from Hogan during the first week of school (at the latest.)

Grade: Homework Assignment worth 20 points. If not completed by 3PM on 9/4.2020, 2 points will be deducted. Another 2 points will be deducted at the 3PM each day thereafter that the assignment is not complete.

E-mail and questions or concerns to Ms. Hogan at smhogan@philasd.org.

Name: _____

DAY ONE: Write the equation of a line.
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Basic Steps:

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| <ol style="list-style-type: none">(1) Find slope(2) Plug slope into $y = mx + b$ for m(3) Plug in point (x, y) and solve for b(4) Rewrite final answer as $y = mx + b$ with correct numerical values for m and b |
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Practice:

1. Write the equation of a line through the two points in slope-intercept form.

- a. $(2, 3)$ and $(4, 2)$

- b. $(-3, 10)$ and $(2, 5)$

- c. $\left(\frac{1}{2}, 5\right)$ and $\left(3, \frac{1}{3}\right)$

- d. $\left(\frac{3}{4}, 1\right)$ and $\left(\frac{5}{4}, 3\right)$

2. Write the equation of a line with the given slope and through the given point in slope-intercept form.

- a. $m = 2$ and $(3, 4)$

- b. $m = -4$ and $(-7, 1)$

c. $m = -1/5$ and $(2, 6)$

d. $m = 3/4$ and $(-10, 2)$

3. Write the equation of a line parallel to the given line through the given point.

a. Line: $y = 3x - 2$ and $(-5, 1)$

b. Line: $y = \frac{1}{3}x + 10$ and $(2, -3)$

c. Line: $4x + 2y = 7$ and $(10, 4)$

d. Line: $-2x - 5y = 3$ and $(4, -1)$

4. Write the equation of a line perpendicular to the given line through the given point.

a. Line: $y = 2x + 8$ and $(6, 5)$

b. Line: $y = \frac{2}{7}x + 5$ and $(3, 1)$

c. Line: $3x - 2y = 8$ and $(-7, 0)$

d. Line: $-3x + 4y = 10$ and $(9, -2)$

DAY TWO: Factor polynomial expressions.**Basic Factoring Guidelines:**

- (1) Factor out a GCF
- (2) Quadratic Trinomial? Try guess-and-check or product-sum to factor into two binomials.
- (3) Even number of terms (more than 2)? Try grouping.
- (4) Binomial? See if it's a difference of squares, sum of cubes, or difference of cubes
- (5) Still striking out? Use synthetic division and educated guesses to factor.

Practice: Fully factor each expression.

1. $5x^3 + 15x^2$

2. $2x^2 + 2x$

3. $2xy - 2x^2y^2$

4. $6x - 9x^2$

5. $100x - 30y$

6. $30x^2 - 40x$

7. $x^3 - x^2$

8. $3 - 9x$

9. $8x + 10$

10. $x^2 + 8x + 7$

11. $x^2 - 11x + 10$

12. $x^2 + x - 90$

13. $x^2 + 4x - 12$

14. $x^2 + 2x - 24$

15. $x^2 - 13x + 40$

16. $x^2 + 11x + 18$

17. $x^2 - x - 56$

18. $2x^2 + 6x - 108$

19. $5x^2 + 10x + 20$

20. $5x^2 - 30x + 40$

21. $4x^2 - 4x - 8$

22. $3x^2 - 2x - 5$

23. $2x^2 + 3x - 9$

24. $3x^2 - 8x + 4$

25. $5x^2 + 19x + 12$

26. $2x^2 + 11x + 5$

27. $2x^2 + 5x + 2$

28. $7x^2 + 53x + 28$

29. $9x^2 + 66x + 21$

30. $15x^2 - 27x - 6$

31. $5x^2 - 18x + 9$

32. $4x^2 - 15x - 25$

33. $6x^2 + 37x + 6$

34. $16x^2 - 9$

35. $4x^2 - 25$

36. $45x^2 - 5$

37. $x^2 - 25$

38. $x^4 - 16$

39. $8x^3 - 1$

40. $27x^3 + 64$

41. $x^3 - 8$

42. $64x^3 + 1$

43. $15x^4 + 120x$

44. $8x^3 - 64x^2 + x - 8$

45. $12x^3 + 2x^2 - 30x - 5$

46. $25x^3 + 5x^2 + 30x + 6$

47. $4x^3 - 12x^2 - 5x + 15$

48. $12x^3 - 21x^2 + 28x - 49$

49. $9x^3 - 27x^2 - 4x + 12$

50. $x^3 - 4x^2 + x + 6$

51. $3x^3 + 8x^2 - x - 10$

DAY THREE: Find the zeros of polynomial functions and solve polynomials equations.
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Basic Guidelines:

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| (1) Make one side of the equation equal zero.
(2) Fully factor the other side of the equation using the steps above.
(3) Set each factor equal to zero and solve each new equation. |
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Part I: Find the zeros of the function without a calculator.

1. $f(x) = x^2 - 5x + 6$

2. $f(x) = 2x^2 - 5x + 3$

3. $f(x) = x^3 - 7x^2 + 6x$

4. $f(x) = x^2 + 3x - 18$

5. $f(x) = 2x^3 + 2x^2 - 4x$

6. $f(x) = 5x^2 - 30x + 40$

7. $f(x) = x^2 - x - 90$

8. $f(x) = x^3 - 64$

9. $f(x) = 81x^4 - 16$

10. $f(x) = 72x^4 - 9x$

11. $f(x) = 24x^3 - 64x^2 - 21x + 56$

12. $f(x) = x^3 - 5x^2 - 9x + 45$

13. $f(x) = 8x^3 + 27$

14. $f(x) = 2x^3 - 11x^2 - x + 30$

Part II: Solve the equation without the use of a calculator.

16. $-7x = x^3 + 8x^2$

17. $5 = x^2 - 4x$

18. $5x = 2x^2 + 3$

19. $9x = 10 - x^2$

20. $12x^3 - 30x = 5 - 2x^2$

21. $4x^4 + 15x = 12x^3 + 5x^2$

<p>DAY FOUR: Simplify rational expressions.</p>

<p>Basic Guidelines:</p>

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| <p>(1) One fraction is always better than multiple fractions.</p> <ol style="list-style-type: none"> Sum or difference of fractions? Get the lowest possible common denominator and combine them into a single fraction. Multiplication of two fractions? Multiply straight across. Factor and cancel. Division of two fractions? Change it to multiplication by the reciprocal of the divisor fraction and then follow the steps for multiplication. A negative exponent means “Hey, I’m on the wrong floor of my own individual fraction.” You will need to rewrite these expressions with positive exponents, find a common denominator and proceed accordingly. <p>(2) Once you have a single fraction, factor the entire numerator and the entire denominator. Factors that appear in both the numerator and the denominator can be cancelled.</p> |
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Practice: Fully simplify each rational expression.

1. $\frac{3}{x-5} + \frac{x}{x+1}$

2. $\frac{3}{x^2+3x+2} - \frac{2}{x^2-1}$

3. $\frac{3x^2-3}{5x^2-65x+60} \cdot \frac{x^3-2x^2-120x}{4x^3}$

4. $\frac{\frac{x^3+64}{45-5x^2}}{-3x^3+12x^2-48x} \cdot \frac{1}{x^2+5x-24}$

$$5. \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h}$$

$$6. \frac{15}{x^2} \cdot \frac{x^5}{12} \cdot \frac{4}{x}$$

$$7. \frac{x^2 - 4x - 5}{x^2 - 3x + 2} \cdot \frac{x^2 - 4}{x^2 - 3x - 10}$$

$$8. \frac{2x^2 - 2x}{x^2 - 9} + \frac{x^2 + x - 2}{x^2 + 2x - 3}$$

$$9. \frac{\frac{x^2 - 9x + 14}{x^2 - 6x + 5}}{\frac{x^2 - 8x + 7}{x^2 + 7x + 10}}$$

$$10. \frac{x}{x-3} - \frac{3}{x+4} + \frac{7}{x^2+x-12}$$

$$11. \frac{2x}{x+3} - \frac{x-3}{x^2+6x+9}$$

$$12. \frac{24}{x^3} \cdot \frac{x^7}{15} \cdot \frac{4}{2x^2}$$

$$13. \frac{x^2+6x}{10} \cdot \frac{4}{x^2-36}$$

$$14. \frac{4x+8}{5x-20} + \frac{x^2+3x-10}{x^2-4x}$$

$$15. \frac{\frac{x^3}{3x^2-12}}{\frac{x^3+5x^2}{3x^2+9x-30}}$$

$$16. \frac{4}{x^2-4} + \frac{x+3}{x+2}$$

$$17. \frac{x-37}{x^2-2x-15} - \frac{5}{x+3}$$

DAY FIVE: Use a calculator to graph functions and to find extrema, zeros, and the intersection of two functions.

Basic Guidelines:

- (1) Enter function into y=
- (2) Be sure that all PLOTS are off and that all unnecessary equations are off as well.
- (3) Select a standard viewing window by using ZOOM 6.
- (4) Graph
- (5) If your window does not show all requisite information (extrema, zeros, intersections) press WINDOW and adjust accordingly
- (6) Use 2ND CALC to choose the appropriate value that you wish to find
 - a. For each MAX, MIN, and ZERO: choose a point to the left (ENT), to the right (ENT), and near the value you want (ENT). Note: for ZERO, this will mean either that the left point is above the axis and the right is below OR that the left point is below the axis and the right is above the axis
 - b. For each INTERSECT: scroll to the point of intersection, hit ENT, ENT, ENT

Practice: Use your calculator to find the zeros and extrema of each function. Write EXACTLY what you entered into the Y= section of your calculator and draw a rough sketch of what you see on your calculator graph.

1. $f(x) = \sqrt[3]{5x-6}$

2. $f(x) = 3x^5 - 4x^3 - 3x^2 + 2x + 1$

3. $f(x) = x^3 - x + 2$

4. $f(x) = x^2 \sin x - 2, [-2\pi, 2\pi]$

5. $f(x) = 5 - \sqrt{10x^2 - 3x + 1}$

6. $f(x) = \frac{2x^3 - 7x}{x^2 + 1}$

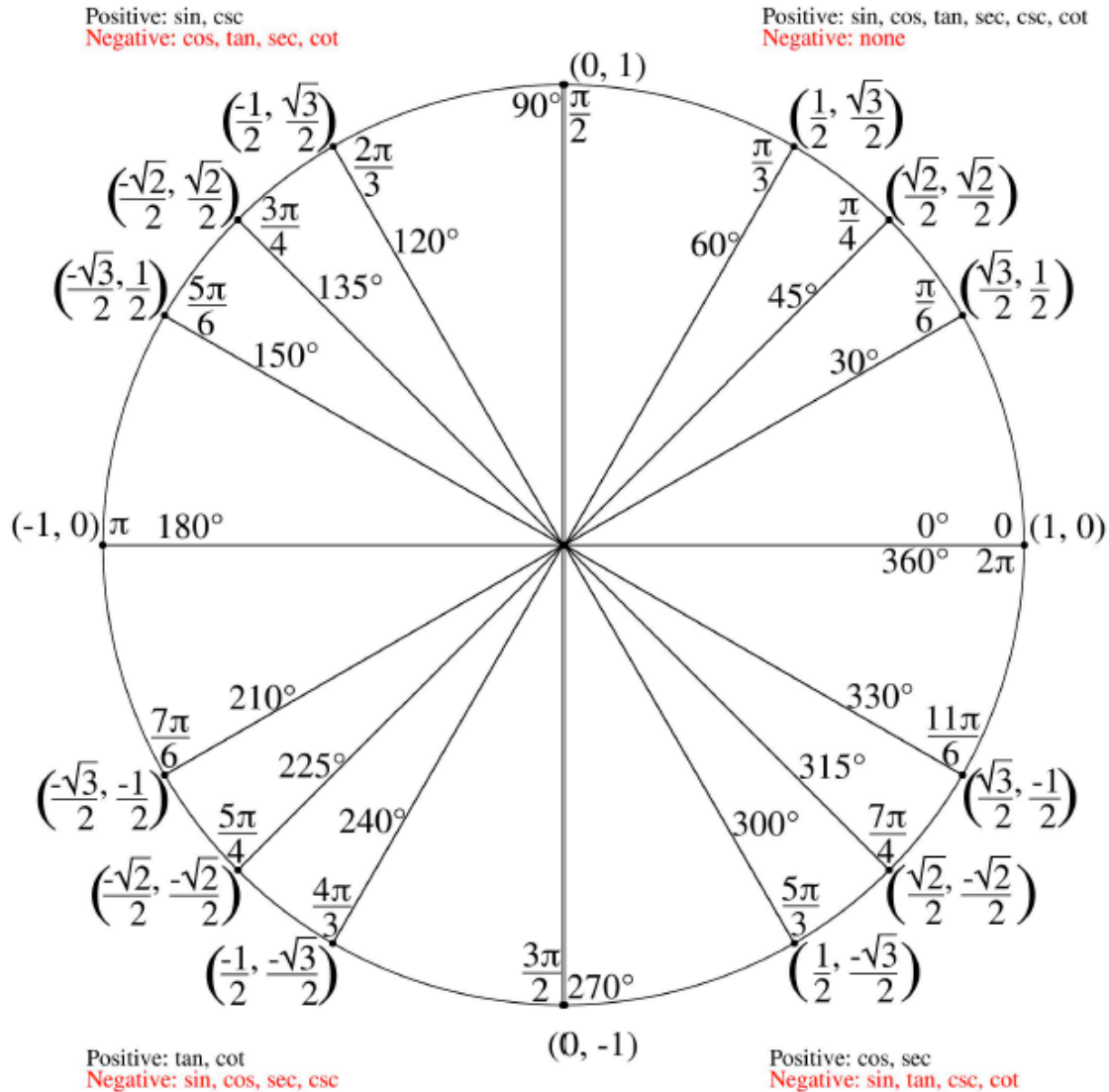
7. $f(x) = \log_2(x - 5)$

8. $f(x) = e^{\frac{x}{2}} \sqrt{x-2} - 5$

DAY SIX: Use the unit circle.

Basic Guidelines:

- (1) Memorize the points in the first quadrant and use your knowledge of the Cartesian coordinate system to understand the other three quadrants. Each point is (Cosine, Sine) for the angle at that location.
- (2) Memorize the degree and radian values for the entire circle.
- (3) Positive angles are created by rotating counterclockwise from the positive x-axis. Negative angles are created by rotating clockwise from the positive x-axis.



Part I: Using the unit circle find the indicated value.

QUESTION	POINT (from circle)	ANSWER
1. $\sin 30^\circ$		
2. $\sin 330^\circ$		
3. $\sin\left(\frac{3\pi}{4}\right)$		
4. $\sin\left(\frac{-2\pi}{3}\right)$		
5. $\cos 225^\circ$		
6. $\cos(-210^\circ)$		
7. $\cos\left(\frac{\pi}{2}\right)$		
8. $\cos\left(\frac{-5\pi}{3}\right)$		
9. $\tan 60^\circ$		
10. $\tan\left(\frac{5\pi}{6}\right)$		
11. $\tan\left(\frac{-3\pi}{2}\right)$		

12. $\cot 270^\circ$		
13. $\cot 315^\circ$		
14. $\cot\left(\frac{-\pi}{6}\right)$		
15. $\sec(-120^\circ)$		
16. $\sec\left(\frac{3\pi}{4}\right)$		
17. $\sec(\pi)$		
18. $\csc 210^\circ$		
19. $\csc\left(\frac{-5\pi}{4}\right)$		
20. $\csc 240^\circ$		

Part II: Using the unit circle, find the indicated angle on the interval $[0^\circ, 360^\circ]$.

QUESTION	POINTS (from circle)	ANGLE
21. $\sin^{-1}\left(\frac{1}{2}\right)$		
22. $\arcsin(0)$		
23. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$		
24. $\arccos(-1)$		
25. $\tan^{-1}(1)$		
26. $\arctan(\sqrt{3})$		
27. $\cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$		
28. $\cot^{-1}(-1)$		
29. $\csc^{-1}(2)$		
30. $\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$		

Part III: Using the unit circle, find the indicated angle on the interval $[0, 2\pi]$.

QUESTION	POINTS (from circle)	ANSWER
31. $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$		
32. $\arcsin(-1)$		
33. $\cos^{-1}\left(\frac{-1}{2}\right)$		
34. $\arccos\left(\frac{\sqrt{3}}{2}\right)$		
35. $\tan^{-1}(0)$		
36. $\arctan\left(\frac{-\sqrt{3}}{3}\right)$		
37. $\cot^{-1}(\sqrt{3})$		
38. $\cot^{-1}(1)$		
39. $\csc^{-1}(-\sqrt{2})$		
40. $\sec^{-1}(2)$		

DAY SEVEN: Factor trigonometric expressions**Basic Guidelines:**

- (1) Generally, you will need to use trig IDs to simplify the expression so that the following statements are true
 - a. The inside of all of the trig functions is the same (for example, all of them have an x or all of them have a $3x$.)
 - b. Often, especially when there are both linear and quadratic powers of various trig functions, it is helpful to change all of the trig functions to the same one by use of Pythagorean IDs.
- (2) Follow all of the basic rules of factoring (see DAY TWO). Be especially conscious of “secret quadratic trinomials.”

1. $\sin x \cos x - \sin x$

2. $2 \sin x \cos x - \cos x$

3. $\sin^3 x \cos^3 x + 4 \sin x \cos^2 x$

4. $\sin^3 x - 4 \cos^2 x$

5. $\sin 2x + \sqrt{3} \cos x$

6. $\tan x \sin x - 2 \sin x$

7. $2 \sin^2 x + 7 \sin x + 3$

8. $8 \cos^3 x + 1$

9. $\tan^2 x + 5 \tan x + 6$

10. $\sin^2 x - 2 \cos x + 7$

11. $16 \sin^4 x - 1$

12. $2 \cos^2 x + \cos x - 1$

13. $\cos 2x - \cos x$

14. $\sec^2 x - 2 \sec x \tan x - \tan^2 x$

15. $6 \csc^2 x - 10 \csc x \sin x - 4 \sin^2 x$

16. $9 \cos^2 x - 3 \cos x \cot x - 2 \cot^2 x$

DAY EIGHT: Find the zeros of trigonometric functions and solve trigonometric equations.

Basic Guidelines:

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| <ol style="list-style-type: none">(1) Make one side of the equation equal zero.(2) Follow all of the guidelines for factoring trigonometric expressions.(3) Set each factor equal to zero and solve using the unit circle. If the value does not appear on the unit circle, use a calculator. Find all values in the designated interval. |
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Part I: Find the zeros of each function on the given interval without a calculator. $[0, 2\pi]$

1. $f(x) = 2 \sin x \tan x - \sqrt{2} \tan x$

2. $f(x) = \csc x \cos x - 2 \cos x$

3. $f(x) = \sin 2x + \sqrt{3} \sin x$

4. $f(x) = \sin^2 x - 3 \sin x + 2$

5. $f(x) = 2 \cos^3 x + 3 \cos^2 x + \cos x$

6. $f(x) = -\cos 2x - \sin x$

DAY NINE: Factoring and simplifying exponential, logarithmic, and radical expressions.
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Basic Factoring Guidelines:

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| <ul style="list-style-type: none"> (1) Exponential Expressions <ul style="list-style-type: none"> a. All exponential must have a same base in order to factor b. Look for “secret quadratic trinomials” with a base to the $2x$ as the squared term and a base to the x as the linear term. (2) Logarithmic Expressions <ul style="list-style-type: none"> a. All logarithms must have the same base in order to factor b. Look for a “secret quadratic trinomial” (3) Radical Expressions <ul style="list-style-type: none"> a. Rewrite all roots as fractional powers b. Look to factor out a GCF (this means that you will subtract that power from all of the other powers) c. Look for “secret quadratic trinomials” which a “squared term” that has a power twice that of a “linear term” |
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Basic Simplifying Guidelines:

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| <ul style="list-style-type: none"> (1) Exponential Expressions <ul style="list-style-type: none"> a. $b^m b^n = b^{m+n}$ b. $(b^m)^n = b^{mn}$ c. $b^m/b^n = b^{m-n}$ (2) Logarithmic Expressions <ul style="list-style-type: none"> a. $\log_b x + \log_b y = \log_b(xy)$ b. $\log_b x - \log_b y = \log_b(x/y)$ c. $P\log_b x = \log_b x^P$ (3) Radical Expressions <ul style="list-style-type: none"> a. Rewrite radicals as fractional exponents b. Follow guidelines for exponents |
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Part I: Fully factor each expression.

1. $2e^{2x} + 5e^x + 3$

2. $5e^{2x} + 30e^x$

3. $6e^x - 18e^x \ln 2x$

4. $4x^{\frac{7}{3}} - 3x^{\frac{4}{3}} - x^{\frac{1}{3}}$

Part II: Expand each logarithmic expression fully.

6. $\log_5 x^2 y^3$

7. $\log_9 \frac{y^4}{x^2 z^5}$

8. $\ln \sqrt[3]{y^2(y+5)}$

Part III: Condense each expression into a single logarithm.

9. $\ln 10 + \ln x$

10. $\log_2 x - 3 \log_2 y - 5 \log_2 z$

11. $\ln x - 3[\ln(x-7) + \ln(x+6)]$

12. $\frac{1}{4}[\log_3 y + 5 \log_3(y-2)] - \log_3(y+4)$

DAY TEN: Find the zeros of or solve exponential, logarithmic and radical functions.

Basic Guidelines: (Make one side of the equation equal zero)

(1) Exponential Equations

- a. Factor as needed.
- b. Once you have a single exponential expression on each side, you can take the logarithm of that base to solve for the exponent.
- c. Once you have solved for the exponent, continue to solve for the variable.

(2) Logarithmic Equations

- a. Combine multiple logarithms into a single logarithm on each side of the equation by using logarithmic properties.
- b. Exponentiate with the appropriate base to get rid of the logarithms.
- c. Solve for the variable.
- d. Check for extraneous answers.

(3) Radical Equations

- a. Factor
- b. Set factors equal to zero and solve

Part I: Find the zeros of each exponential, logarithmic, or radical function without a calculator.

1. $f(x) = 10e^{2x} - 20e^x$

2. $f(x) = 2e^{2x} - 7e^x + 3$

3. $f(x) = 3e^{3x} - 12e^x$

4. $f(x) = \log_4(2x + 10) - \log_4(4 - x) - 2$

5. $f(x) = \log_2(x-5) + \log_2(x+2) - 3$

6. $f(x) = 5^{2x} - 3 \cdot 5^x + 2$

7. $f(x) = e^{2x} + e^x - 20$

8. $f(x) = \sqrt[4]{x^{11}} - 5\sqrt[4]{x^7} + 6\sqrt[4]{x^3}$

Part II: Solve each exponential, logarithmic, or radical function without a calculator.

9. $8 \ln 2x = 5$

10. $\log_2 x - \log_2 7 = 3$

11. $\ln \sqrt{x-3} = 2$

<p>DAY ELEVEN: Determine the domain (inputs) and range (outputs) of a function. Program your calculator with useful programs.</p>			
<p>Basic Domain Guidelines:</p> <p>(1) Visually, the domain is all x-values for which there exists a curve. It excludes:</p> <ol style="list-style-type: none"> Vertical Asymptotes Holes <p>(2) Algebraically, the domain of a function (without any listed domain constraints) is all real numbers EXCEPT for the BIG FOUR exceptions:</p> <ol style="list-style-type: none"> Any x-value that makes the denominator zero (you can't divide by zero) Any x-value that makes the inside of an even root negative Any x-value that makes the inside of a logarithm zero or negative Any real world constraints (only an issue in word problems) 			
<p>Basic Range Guidelines:</p> <p>(1) Visually, the range is all y-values for which there exists a curve. It CAN exclude:</p> <ol style="list-style-type: none"> Horizontal Asymptotes Holes <p>(2) Algebraically, the range can be figured out using a basic knowledge of several common graphs. A few examples (but not a comprehensive list) are:</p> <ol style="list-style-type: none"> Polynomials of EVEN degree <ol style="list-style-type: none"> Positive leading coefficient (smile) has a range from the absolute minimum y-value to infinity. Negative leading coefficient (frown) has a range from negative infinity to the absolute maximum y-value. Polynomials of an ODD degree have a range of all real numbers No negative y-values can come out of an EVEN root Sin(x) and Cos(x) both have a range of [-1, 1] Sec(x) and Csc(x) both have range of (neg. infinity, -1] U [1, infinity) Tan(x) and Cot(x) both have range of all real numbers No negative y-values can come out of an absolute value 			
<p>Basic Programming Guidelines:</p> <ol style="list-style-type: none"> Name your program something useful and specific CLR HOME PROMPT for the values you know Solve the equation for the value that you want and DISP (Display) the resulting equation 			

DOMAIN AND RANGE PRACTICE:

	FUNCTION	DOMAIN	RANGE
1	$f(x) = x^2 - 5$		
2	$f(x) = 2x^3 + 7x - 4$		

3	$f(x) = 13 - 5x^2$		
4	$f(x) = 3\sin x$		
5	$f(x) = \sqrt{x-6}$		
6	$f(x) = \sqrt[3]{2x-9}$		
7	$f(x) = \ln(x^2 + 3)$		