George Washington Carver
High School of Engineering and Science
2019 Summer Enrichment

PRECALCULUS—Due Wednesday September 11th

If you are taking BOTH Algebra II AND Precalculus in the fall of 2019, you will only need to complete the Algebra II summer project this year.

Name: _____ Advisory #: _____

YOUR PROJECT WILL BE GRADED BASED ON THIS RUBRIC.

Guideline	Points Earned	Points Possible
Bottle Drainage Experiment (40 points total)		
Data Table (6 points total)		
Five Second Increments		2
Units for X		2
Units for Y		2
Scatter Plot (18 points total)		
Label X-Axis		2
Label Y-Axis		2
Scale X-Axis		2
Scale Y-Axis		2
All points plotted correctly		8
Title		2
Quadratic Regression (4 points total)		
In the form $Y = Ax^2+Bx+c$		2
A, B, and C are constants		2
Observations (12 points total)		
Domain (should be an interval, not a list of values)		2
Range (should be an interval, not a list of values)		2
Good or bad fit?		2
Specific reasons for good or bad fit.		2
How does the size of the hole alter the experiment? Explain.		2
Other Observations:		2
Algebra Review Work (60 points total)		
Topic A (#1-20) Answers and sufficient work.		12
Topic B (#21-34) Answers and sufficient work.		12
Topic C (#35-59) Answers and sufficient work.		12
Topic D (#60-74) Answers and sufficient work.		12
Topic E (#75-84) Answers and sufficient work.		12
TOTALS		100

Quadratic Model for Bottle Drainage

Objective: To observe and model how the level of a draining container decreases. This project will provide the opportunity to draw connections between quadratic equations and real-world phenomena. You will need a family member or friend to help you with the experiment. For convenience, do the experiment in a kitchen.

Materials needed: a plastic bottle, water proof tape, a watch with a second hand, a metric ruler (centimeters), a black marker

Experiment: Take the empty plastic bottle and punch a hole into the bottom of the bottle. Cover the hole with the tape and fill the bottle to the top with water. Mark the fill level on the outside of the bottle with the marker. Open the hole and allow the water to drain into a sink, bucket or other container. As the water drains, mark the level of the water on the bottle every five seconds. Use your metric ruler to measure the water level in centimeters at 0 seconds, 5 seconds, 10 seconds and so on.

After all of the water is drained from the bottle, read and follow the steps below to write your report. Your written report should be one to two pages in length.

1. Data Table Make a data table with five second time increments in one column, and the water level measured in centimeters in the second column. (a table will be provided on p. 2)

2. Make a scatter plot form the data table: Use graph paper to make a scatter plot of your data. (graph paper provided on p. 3)

3. Quadratic Regression Equation: Enter the data from the data table into two lists on your graphing calculator. Calculate the quadratic regression and write the equation below the scatter plot.

4. Observations: What are the domain and range? In what ways is a quadratic model a good fit for this data? In what ways is a quadratic model not a good fit? How would the scatter plot be different if the hole was smaller? Larger? Explain.

5. Other observations: Describe additional observations that you can make. Be specific.

Data Table:

Observations:

What are the domain and range?

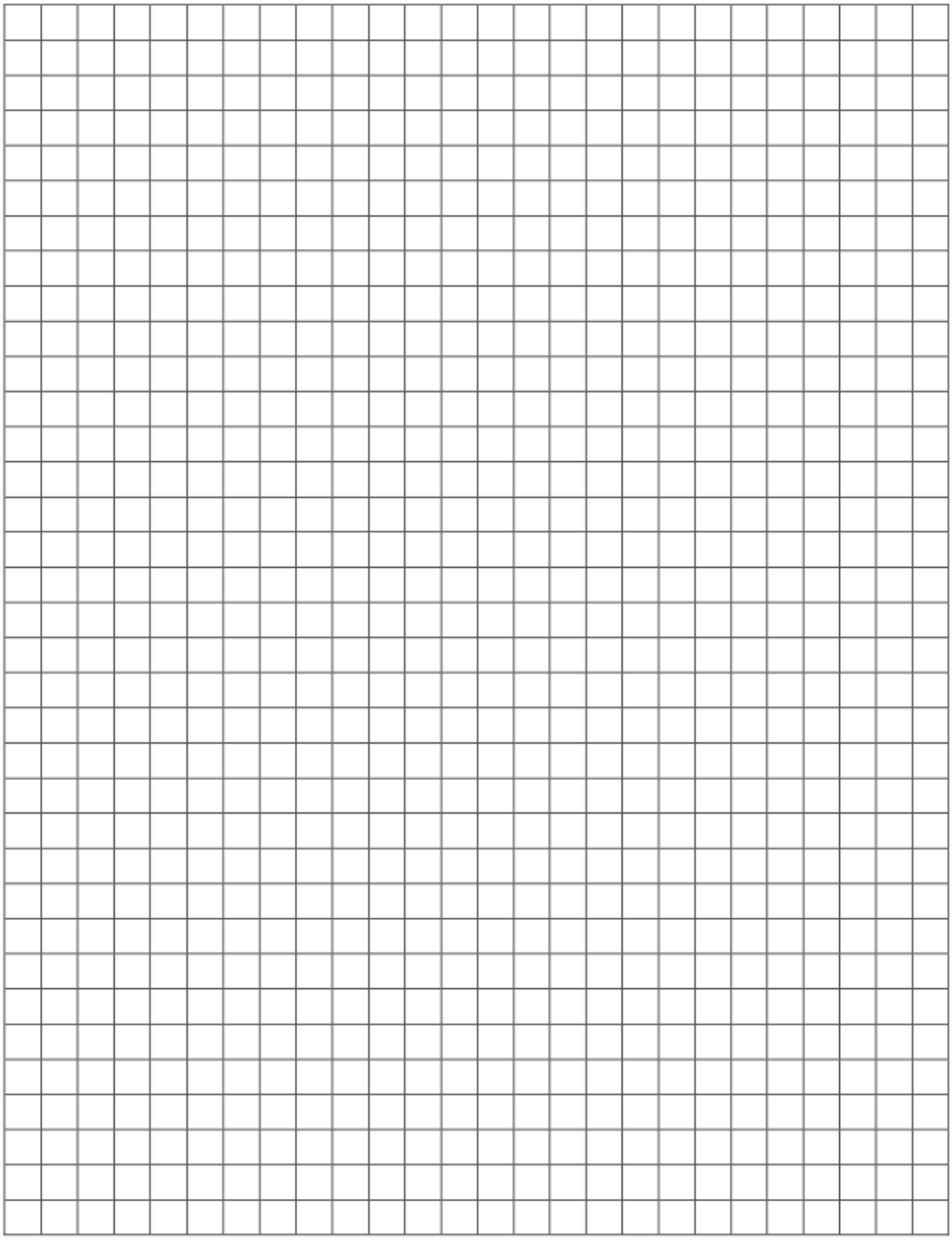
Is the quadratic model a good fit or a bad fit for this data? Give specific reasons for your answer.

How would the scatter plot be different if the hole was smaller? Larger? Explain.

Other observations? Would you do anything differently with this experiment if you did it again?

Scatter Plot:

Title:_____



Quadratic Regression Equation:

A. <u>Algebraic operations, grouping, evaluation</u>:
To evaluate an expression, first calculate the powers, then multiply and divide in order from left to right, and finally add and subtract in order from left to right. Parentheses have preference.

example: $14-3^2=14-9=5$
example: $2\bullet 4+3\bullet 5=8+15=23$
example: $10-2\bullet 3^2=10-2\bullet 9=10-18=-8$
example: $(10-2)\bullet 3^2=8\bullet 9=72$

Problems 1-7: Find the value:

1. $2^3=$
2. $-2^4=$
3. $4+2\bullet 5=$
4. $3^2-2\bullet 3+1=$
5. $0^4=$
6. $(-2)^4=$
7. $1^5=$

Problems 8-13: Find the value if $a=-3$, $b=2$, $c=0, d=1$, and $e=-3$:

8. $a-e=$
9. $e^2+(d-ab)c=$
10. $a-(bc-d)+e=$
11. $\frac{e}{d}+\frac{b}{a}-\frac{2d}{e}=$
12. $\frac{b}{e}=$
13. $\frac{d}{c}=$

Combine like terms when possible:

example: $3x+y^2-(x+2y^2)$
$=3x-x+y^2-2y^2=2x-y^2$
example: $a-a^2+a=2a-a^2$

Problems 14-20: Simplify:

14. $6x+3-x-7=$
15. $2(3-t)=$
16. $10r-5(2r-3y)=$
17. $x^2-(x-x^2)=$
18. $3a-2\big(4(a-2b)-3a\big)$
19. $3(a+b)-2(a-b)=$
20. $1+x-2x+3x-4x=$

#	Answer and Sufficient Work (Attached papers if need)
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B. Simplifying fractional expressions:

example: $\frac{27}{36} = \frac{9\cdot 3}{9\cdot 4} = \frac{9}{9}\cdot\frac{3}{4} = 1\cdot\frac{3}{4} = \frac{3}{4}$
(note that you must be able to find a common factor - in this case 9 - in both the top and bottom in order to reduce a fraction.)

example: $\frac{3a}{12ab} = \frac{3a\cdot 1}{3a\cdot 4b} = \frac{3a}{3a}\cdot\frac{1}{4b} = 1\cdot\frac{1}{4b} = \frac{1}{4b}$
(common factor: $3a$)

Problems 21-32: Reduce:

21. $\frac{13}{52} =$

22. $\frac{26}{65} =$

23. $\frac{3+6}{3+9} =$

24. $\frac{6axy}{15by} =$

25. $\frac{19a^2}{95a} =$

26. $\frac{14x-7y}{7y} =$

27. $\frac{5a+b}{5a+c} =$

28. $\frac{x-4}{4-x} =$

29. $\frac{2(x+4)(x-5)}{(x-5)(x-4)} =$

30. $\frac{x^2-9x}{x-9} =$

31. $\frac{8(x-1)^2}{6(x^2-1)} =$

32. $\frac{2x^2-x-1}{x^2-2x+1} =$

example: $\frac{3}{x}\cdot\frac{y}{15}\cdot\frac{10x}{y^2} = \frac{3\cdot 10\cdot x\cdot y}{15\cdot x\cdot y^2} =$

$\frac{3}{3}\cdot\frac{5}{5}\cdot\frac{2}{1}\cdot\frac{x}{x}\cdot\frac{y}{y}\cdot\frac{1}{y} =$

$1\cdot 1\cdot 2\cdot 1\cdot 1\cdot\frac{1}{y} = \frac{2}{y}$

Problems 33-34: Simplify:

33. $\frac{4x}{6}\cdot\frac{xy}{y^2}\cdot\frac{3y}{2} =$

34. $\frac{x^2-3x}{x-4}\cdot\frac{x(x-4)}{2x-6} =$

#	Answer and Sufficient Work (Attached papers if need)
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C. Laws of integer exponents:

I. $a^b \bullet a^c = a^{b+c}$

II. $\frac{a^b}{a^c} = a^{b-c}$

III. $(a^b)^c = a^{bc}$

IV. $(ab)^c = a^c \bullet b^c$

V. $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

VI. $a^0 = 1$ (if $a \neq 0$)

VII. $a^{-b} = \frac{1}{a^b}$

Problems 35-44: Find x:

35. $2^3 \bullet 2^4 = 2^x$
36. $\frac{2^3}{2^4} = 2^x$
37. $3^{-4} = \frac{1}{3^x}$
38. $\frac{5^2}{5^2} = 5^x$
39. $\left(2^4\right)^3 = 2^x$
40. $8 = 2^x$
41. $a^x = a^3 \bullet a$
42. $\frac{b^{10}}{b^5} = b^x$
43. $\frac{1}{c^{-4}} = c^x$
44. $\frac{a^{3y-2}}{a^{2y-3}} = a^x$

Problems 45-59: Simplify:

45. $8x^0 =$
46. $3^{-4} =$
47. $2^3 \bullet 2^4 =$
48. $0^5 =$
49. $5^0 =$
50. $(-3)^3 - 3^3 =$
51. $2^x \bullet 4^{x-1} =$
52. $\frac{2^{c+3}}{2^{c-3}} =$
53. $2^{c+3} \bullet 2^{c-3} =$
54. $\frac{8^x}{2^{x-1}} =$

55. $\frac{2x^{-3}}{6x^{-4}} =$
56. $\left(a^{x+3}\right)^x =$
57. $\frac{a^{3x-2}}{a^{2x-3}} =$
58. $\left(-2a^2\right)^4\left(ab^2\right) =$
59. $2\left(4xy^2\right)^{-1}\left(-2x^{-1}y\right)^2 =$

#	Answer and Sufficient Work (Attached papers if need)
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D. Scientific notation:

example: $32800 = 3.2800 \times 10^4$ if the zeros i
the ten's and one's places are significant. If the one's zero is not, write 3.280×10^4; if neither is significant: 3.28×10^4

example: $.004031 = 4.031 \times 10^{-3}$

example: $2 \times 10^2 = 200$

example: $9.9 \times 10^{-1} = .99$

Note that scientific form always looks like $a \times 1$ where $1 \le a < 10$, and n is an integer power of 1

Problems 60-63: Write in scientific notation:

60. 93,000,000 =
61. .000042 =
62. 5.07 =
63. -32 =

Problems 64-66: Write in standard notation:

64. 1.4030×10^3 =
65. -9.11×10^{-2} =
66. 4×10^{-6} =

To compute with numbers written in scientific f
separate the parts, compute, and then recombine

example: $(3.14 \times 10^5)(2) = (3.14)(2) \times 10^5$
$= 6.28 \times 10^5$

example: $\frac{4.28 \times 10^6}{2.14 \times 10^{-2}} = \frac{4.28}{2.14} \times \frac{10^6}{10^{-2}} = 2.00 \times 10^8$

example: $\frac{2.01 \times 10^{-3}}{8.04 \times 10^{-6}} = .250 \times 10^3 = 2.50 \times 10^2$

Problems 67-74: Write answer in scientific notati

67. $10^{40} \times 10^{-2}$ =
68. $\frac{10^{-40}}{10^{-10}}$ =
69. $\frac{1.86 \times 10^4}{3 \times 10^{-1}}$ =
70. $\frac{3.6 \times 10^{-5}}{1.8 \times 10^{-8}}$ =
71. $\frac{1.8 \times 10^{-8}}{3.6 \times 10^{-5}}$ =
72. $(4 \times 10^{-3})^2$ =
73. $(2.5 \times 10^2)^{-1}$ =
74. $\frac{(-2.92 \times 10^3)(4.1 \times 10^7)}{-8.2 \times 10^{-3}}$ =

#	Answer and Sufficient Work (Attached papers if need)
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E. Absolute value:

example: $|3| = 3$

example: $|-3| = 3$

example: $|a|$ depends on a

if $a \geq 0$, $|a| = a$

if $a < 0$, $|a| = -a$

example: $-|-3| = -3$

Problems 75-78: Find the value:

75. $|0| =$

76. $\frac{|a|}{a} =$

77. $|3| + |-3| =$

78. $|3| - |-3| =$

Problems 79-84: If $x = -4$, find:

79. $|x + 1| =$

80. $|1 - x| =$

81. $-|x| =$

82. $x + |x| =$

83. $|-3x| =$

84. $|\,(x - |(x - |x|\,)|\,)| =$

#	Answer and Sufficient Work (Attached papers if need)
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